The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

LaBRI, LIAFA

June 25, 2012
• The PASEP: particles on a one-dimensional lattice
• The PASEP: particles on a one-dimensional lattice
• hop to the left/right
The PASEP: particles on a one-dimensional lattice
- hop to the left/right
- enter/exit from the left/right
• The PASEP: particles on a one-dimensional lattice
• hop to the left/right
• enter/exit from the left/right
• a Markov chain
- The PASEP is studied through objects called staircase tableaux
- Labeled tree-like tableaux reveal an underlying tree structure in staircase tableaux
- Combinatorial properties
1 Definitions

Staircase tableaux

Labeled tree-like tableaux

The insertion algorithm

2 Combinatorial applications

\( Z_n(1; \alpha, \beta, \gamma, \delta; 1) \)

\( Z_n(\gamma; 1, 1, 0, 1; 1) \)

A bijective proof for \( Z_n(1; 1, 1, 1, 0; 0) \)
Definition

A staircase tableau of size $n$ is a Ferrers diagram s.t. the $i$-th row is of length $(n - i + 1)$, and that some boxes are labeled with $\alpha, \beta, \gamma$ et $\delta$, satisfying some conditions.
Definition

A staircase tableau of size $n$ is a Ferrers diagram s.t. the $i$-th row is of length $(n - i + 1)$, and that some boxes are labeled with $\alpha$, $\beta$, $\gamma$ et $\delta$, satisfying some conditions.

To fill the remaining boxes

- On the left of a $\beta$ there is a $u$.
- On the left of a $\delta$ there is a $q$.
- Above an $\alpha$ or a delta $\delta$ there is a $u$.
- Above a $\beta$ or a $\gamma$ there is a $q$. 
Definition

A staircase tableau of size $n$ is a Ferrers diagram s.t. the $i$-th row is of length $(n - i + 1)$, and that some boxes are labeled with $\alpha$, $\beta$, $\gamma$ et $\delta$, satisfying some conditions.

To fill the remaining boxes

- On the left of a $\beta$ there is a $u$.
- On the left of a $\delta$ there is a $q$.
- Above an $\alpha$ or a delta $\delta$ there is a $u$.
- Above a $\beta$ or a $\gamma$ there is a $q$. 
Definition

A staircase tableau of size $n$ is a Ferrers diagram s.t. the $i$-th row is of length $(n - i + 1)$, and that some boxes are labeled with $\alpha$, $\beta$, $\gamma$ et $\delta$, satisfying some conditions.

To fill the remaining boxes

- On the left of a $\beta$ there is a $u$.
- On the left of a $\delta$ there is a $q$.
- Above an $\alpha$ or a delta $\delta$ there is a $u$.
- Above a $\beta$ or a $\gamma$ there is a $q$. 
### Definition

A staircase tableau of size \( n \) is a Ferrers diagram s.t. the \( i \)-th row is of length \( (n - i + 1) \), and that some boxes are labeled with \( \alpha, \beta, \gamma \) et \( \delta \), satisfying some conditions.

To fill the remaining boxes:
- On the left of a \( \beta \) there is a \( u \).
- On the left of a \( \delta \) there is a \( q \).
- Above an \( \alpha \) or a delta \( \delta \) there is a \( u \).
- Above a \( \beta \) or a \( \gamma \) there is a \( q \).
Definition

A staircase tableau of size $n$ is a Ferrers diagram s.t. the $i$-th row is of length $(n - i + 1)$, and that some boxes are labeled with $\alpha$, $\beta$, $\gamma$ et $\delta$, satisfying some conditions.
A staircase tableau of size $n$ is a Ferrers diagram s.t. the $i$-th row is of length $(n - i + 1)$, and that some boxes are labeled with $\alpha$, $\beta$, $\gamma$ and $\delta$, satisfying some conditions.

\[ \text{wt}(T) = \alpha^3 \beta^2 \gamma^3 \delta^3 q^9 \]

\[ Z_n(\alpha, \beta, \gamma, \delta, q) = \sum_{T \in T_n} \text{wt}(T) \]

\[ Z_n(y; \alpha, \beta, \gamma, \delta, q) = \sum_{T \in T_n} \text{wt}(T)y^{t(T)} \]
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
1. Staircase tableaux
2. Labeled tree-like tableaux
3. The insertion algorithm

Combinatorial applications
1. \( Z_n(1; \alpha, \beta, \gamma, \delta; 1) \)
2. \( Z_n(y; 1, 1, 0, 1; 1) \)
3. A bijective proof for \( Z_n(1; 1, 1, 1, 0; 0) \)
Labeled tree-like tableaux

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications
\( Z_n(1; \alpha, \beta, \gamma, \delta; 1) \)
\( Z_n(\gamma; 1, 1, 0, 1; 1) \)
A bijective proof for 
\( Z_n(1; 1, 1, 1, 0; 0) \)
Labeled tree-like tableaux

Border edges
Labeled tree-like tableaux

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications
$Z_n(1; \alpha, \beta, \gamma; 1)$
$Z_n(y; 1, 1, 0, 1; 1)$
A bijective proof for
$Z_n(1; 1, 1, 0, 0)$

$L_D(c)$
Labeled tree-like tableaux

\[ Z_n(1; \alpha, \beta, \gamma, \delta; 1) \]
\[ Z_n(\gamma; 1, 1, 0; 1) \]

A bijective proof for
\[ Z_n(1; 1, 1, 0; 0) \]
Labeled tree-like tableaux

Labeled tree-like tableau

A labeled tree-like tableau $T$ of size $n$ is a Ferrers diagram of half-perimeter $n + 1$ such that some boxes and all border edges are labeled with $1$, $\alpha$, $\beta$, $\gamma$, $\delta$, and satisfying the conditions:

- the Northwestern-most box (root box) is labeled by $1$;
- the labels in the first row and the first column are the only labels $1$;
- in each row and column, there is at least one labeled box;
- for each label $\alpha$ or $\gamma$, all boxes in $L_T(c)$ are empty and at least one box in $A_T(c)$ is labeled;
- for each label $\beta$ or $\delta$, all boxes in $A_T(c)$ are empty and at least one box in $L_T(c)$ is labeled.
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications
$Z_n(1;\alpha,\beta,\gamma,\delta;1)$
$Z_n(\gamma;1,1,0,1;1)$
A bijective proof for
$Z_n(1;1,1,1,0;0)$

A labeled tree-like tableau

```
1 1 1
1 1
1 α
1 β
1 α
```

A labeled tree-like tableau
Bijection

Proposition

For \( n \geq 2 \), staircase tableaux of size \( n - 1 \) are in bijection with LTTLTs of size \( n \).
Proposition

For $n \geq 2$, staircase tableaux of size $n - 1$ are in bijection with LTLTs of size $n$. 
Bijection

Proposition

For $n \geq 2$, staircase tableaux of size $n - 1$ are in bijection with LTLTs of size $n$. 
**Proposition**

For \( n \geq 2 \), staircase tableaux of size \( n - 1 \) are in bijection with LTLTs of size \( n \).
Proposition

For $n \geq 2$, staircase tableaux of size $n - 1$ are in bijection with LTLTs of size $n$. 

\begin{center}
\begin{tikzpicture}
\node[shape=circle,draw=black] (a) at (0,0) {1};
\node[shape=circle,draw=black] (b) at (1,0) {1};
\node[shape=circle,draw=black] (c) at (1,1) {1};
\node[shape=circle,draw=black] (d) at (2,1) {$\alpha$};
\node[shape=circle,draw=black] (e) at (2,2) {$\delta$};
\node[shape=circle,draw=black] (f) at (3,2) {$\alpha$};
\node[shape=circle,draw=black] (g) at (3,3) {$\beta$};
\node[shape=circle,draw=black] (h) at (4,3) {$\gamma$};
\node[shape=circle,draw=black] (i) at (4,4) {$\beta$};
\node[shape=circle,draw=black] (j) at (5,4) {1};
\node[shape=circle,draw=black] (k) at (5,5) {1};
\end{tikzpicture}
\end{center}
Bijection

Proposition

For $n \geq 2$, staircase tableaux of size $n - 1$ are in bijection with LTLTs of size $n$. 

\[
\begin{array}{cccc}
 1 & 1 & 1 \\
 1 & 1 & \alpha \\
 1 & \delta & \alpha \\
 1 & \beta & \gamma \\
 1 & & \\
\end{array}
\leftrightarrow
\begin{array}{cccc}
 1 & 1 & 1 \\
 1 & \alpha & \\
 \delta & \alpha & \\
 \beta & \gamma & \\
 & & \\
\end{array}
\]
1 Definitions

Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

2 Combinatorial applications

$Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
$Z_n(y; 1, 1, 0, 1; 1)$
A bijective proof for $Z_n(1; 1, 1, 1, 0; 0)$
The insertion algorithm induces a bijection between

- LTLTs of size $n$ together with the choice of a border edge and a compatible bi-label
- LTLTs of size $n + 1$
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications
$Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
$Z_n(\gamma; 1, 1, 0; 1)$
A bijective proof for $Z_n(1; 1, 1, 1, 0; 0)$

Row/column addition

$\begin{array}{ccc}
1 & 1 & 1 \\
1 & \alpha & \\
1 & \alpha & \\
1 & \beta & \\
\end{array}$

$(x, y)$
Row/column addition

(\(x, y\))
Definitions
- Staircase tableaux
- Labeled tree-like tableaux

Combinatorial applications
- $Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
- $Z_n(\gamma; 1, 1, 0, 1; 1)$
- A bijective proof for $Z_n(1; 1, 1, 1, 0; 0)$

Ribbon insertion
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
- Staircase tableaux
- Labeled tree-like tableaux

The insertion algorithm

Combinatorial applications

\[ Z_n(1; \alpha, \beta, \gamma, \delta; 1) \]
\[ Z_n(\gamma; 1, 1, 0, 1; 1) \]
A bijective proof for
\[ Z_n(1; 1, 1, 1, 0; 0) \]
The special box

Definition

The special box of a \( \text{LTTLT} \) is the Northeastern-most labeled box among those that occur at the bottom of a column.
The insertion algorithm

**Require** : a LTLT $T$ of size $n$, one of its border edges $e$, a compatible bi-label $(x, y)$.

1. Find the special box $s$ of $T$.
2. Add a row/column to $T$ at edge $e$ with new bi-label $(x, y)$.
3. If $e$ is to the left of $s$, perform a ribbon addition between $e$ and $s$.

**Ensure** : A final LTLT $T'$ of size $n + 1$. 
The insertion algorithm: an example
The insertion algorithm: an example
The insertion algorithm: an example

Definitions
- Staircase tableaux
- Labeled tree-like tableaux

The insertion algorithm

Combinatorial applications
- $Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
- $Z_n(\gamma; 1, 1, 0, 1; 1)$

A bijective proof for
- $Z_n(1; 1, 1, 1, 0; 0)$
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications
$Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
$Z_n(\gamma; 1, 1, 0, 1; 1)$
A bijective proof for $Z_n(1; 1, 1, 1, 0; 0)$

Plan

1. Definitions
   Staircase tableaux
   Labeled tree-like tableaux
   The insertion algorithm

2. Combinatorial applications
   $Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
   $Z_n(\gamma; 1, 1, 0, 1; 1)$
   A bijective proof for $Z_n(1; 1, 1, 1, 0; 0)$
A product formula for
\[ Z_n(1; \alpha, \beta, \gamma, \delta; 1) \]

\[ Z_n(1; \alpha, \beta, \gamma, \delta; 1) = \prod_{j=0}^{n-1} (\alpha + \beta + \gamma + \delta + j(\alpha + \gamma)(\beta + \delta)) \]
Plan

1. Definitions
   - Staircase tableaux
   - Labeled tree-like tableaux
   - The insertion algorithm

2. Combinatorial applications
   - $Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
   - $Z_n(y; 1, 1, 0, 1; 1)$
   - A bijective proof for $Z_n(1; 1, 1, 0, 0)$
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications

$Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
$Z_n(y; 1, 1, 0, 1; 1)$

A bijective proof for
$Z_n(1; 1, 1, 1, 0; 0)$

Study of $Z_n(y; 1, 1, 0, 1; 1)$

Proposition

The polynomial $Z_n(y; 1, 1, 0, 1; 1)$ has all its roots in the segment $]-1, 0[$; it is stable and log-concave.

$T(n, k) = \#\{T \text{ of size } n, \text{ without } \gamma, \text{ having } k \alpha/\delta \text{ on the diagonal}\}$

Let $P_n(y) = \sum_{k \leq n} T(n, k) y^k$
Study of \( Z_n(y;1,1,0,1;1) \)

**Proposition**

The polynomial \( Z_n(y;1,1,0,1;1) \) has all its roots in the segment \([-1,0] \); it is stable and log-concave.

\[
T(n, k) = \#\{T \text{ of size } n, \text{ without } \gamma, \text{ having } k \alpha/\delta \text{ on the diagonal}\}
\]

Let \( P_n(y) = \sum_{k \leq n} T(n, k)y^k \)

Then

\[
P_n(y) = (1 + (n+1)y + (n-1)y^2)P_{n-1}(y) + (y - y^3)P'_{n-1}(y)
\]
Study of $Z_n(y;1,1,0,1;1)$

Proposition

The polynomial $Z_n(y;1,1,0,1;1)$ has all its roots in the segment $]-1,0[$; it is stable and log-concave.

$$T(n,k) = \# \{ \text{T of size } n, \text{ without } \gamma, \text{ having } k \alpha/\delta \text{ on the diagonal} \}$$

Let $P_n(y) = \sum_{k \leq n} T(n,k)y^k$

Then

$$P_n(y) = (1 + (n+1)y + (n-1)y^2)P_{n-1}(y) + (y - y^3)P'_{n-1}(y)$$

Let $Q(y, t) = \sum_n P_n(y) \frac{t^n}{n!}$, then $Q(y, t) = \left( \frac{y-1}{2y-(y+1)e^{t(y-1)}} \right)^{3/2} e^{\frac{t(y-1)}{2}}$
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications
$Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
$Z_n(y; 1, 1, 0, 1; 1)$
A bijective proof for $Z_n(1; 1, 1, 1, 0; 0)$

1 Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

2 Combinatorial applications
$Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
$Z_n(y; 1, 1, 0, 1; 1)$
A bijective proof for $Z_n(1; 1, 1, 1, 0; 0)$
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications
$Z_n(1; 1, 1, 1, 0; 0)$

What is the number of staircase tableaux with no $qs$ nor $\delta$s?
A lazy path

**Definition**

A lazy path of size $n$ is a path on the lattice $\mathbb{Z} \times \mathbb{Z}$

- starting at $(0, 0)$, ending at $(2n, 0)$
- whose steps are $(1, 1)$, $(1, -1)$ or $(2, 0)$
- such that its steps $(2, 0)$ only appear on the $x$-axis
A lazy path of size $n$ is a path on the lattice $\mathbb{Z} \times \mathbb{Z}$

- starting at $(0, 0)$, ending at $(2n, 0)$
- whose steps are $(1, 1), (1, -1)$ or $(2, 0)$
- such that its steps $(2, 0)$ only appear on the x-axis

A lazy path of size 7
From staircase tableaux to trees

\[
\begin{array}{ccc}
\beta & \alpha & \gamma \\
\alpha & \gamma & \\
\beta & \alpha & \\
\beta & & \\
\gamma & & \\
\end{array}
\]
From staircase tableaux to trees

\[ \beta \quad | \\
\alpha/\gamma \quad | \\
\]
From staircase tableaux to trees

Definitions
- Staircase tableaux
- Labeled tree-like tableaux
- The insertion algorithm

Combinatorial applications
- $Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
- $Z_n(\gamma; 1, 1, 0, 1; 1)$
- A bijective proof for $Z_n(1; 1, 1, 1, 0; 0)$
From staircase tableaux to trees
From staircase tableaux to trees

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications
$Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
$Z_n(y; 1, 1, 0, 1; 1)$
A bijective proof for $Z_n(1; 1, 1, 1, 0; 0)$
From staircase tableaux to trees

$q \ X \iff \text{nodes of left depth more than 1}$
From trees to paths
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications

- $Z_n(\gamma; 1, 1, 1, 0; 1)$
- A bijective proof for $Z_n(\gamma; 1, 1, 1, 0; 0)$

From trees to paths
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications
$Z_n(1; \alpha, \beta, \gamma, \delta, 1)$
$Z_n(y; 1, 1, 0, 1, 1)$
A bijective proof for $Z_n(1; 1, 1, 1, 0, 0)$

From trees to paths
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
- Staircase tableaux
- Labeled tree-like tableaux
- The insertion algorithm

Combinatorial applications
- $Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
- $Z_n(\gamma; 1, 1, 0, 1; 1)$
- A bijective proof for $Z_n(1; 1, 1, 1, 0; 0)$

From trees to paths
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications
\[ Z_n(1; \alpha, \beta, \gamma, \delta; 1) \]
\[ Z_n(\gamma; 1, 1, 0; 1) \]
A bijective proof for
\[ Z_n(1; 1, 1, 0, 0) \]

From trees to paths
From trees to paths

Definitions
- Staircase tableaux
- Labeled tree-like tableaux
- The insertion algorithm

Combinatorial applications
- $Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
- $Z_n(\gamma; 1.1.0; 1)$

A bijective proof for $Z_n(1; 1.1.1.0; 0)$
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
- Staircase tableaux
- Labeled tree-like tableaux
- The insertion algorithm

Combinatorial applications

$Z_n(1; \alpha, \beta, \gamma, \delta; 1)$

$Z_n(y; 1, 1, 0, 1; 1)$

A bijective proof for $Z_n(1; 1, 1, 0, 0)$

From trees to paths
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
- Staircase tableaux
- Labeled tree-like tableaux
- The insertion algorithm

Combinatorial applications
- $Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
- $Z_n(\gamma; 1, 1, 0, 1; 1)$
- A bijective proof for $Z_n(1; 1, 1, 1, 0; 0)$

From trees to paths
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications
$Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
$Z_n(\gamma; 1, 1.0, 1; 1)$
A bijective proof for
$Z_n(1; 1.1, 1.0; 0)$

From trees to paths
From trees to paths

The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications
$Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
$Z_n(\gamma; 1, 1, 0, 1; 1)$
A bijective proof for $Z_n(1; 1, 1, 1, 0; 0)$
From trees to paths
From paths to lazy paths
From paths to lazy paths
From paths to lazy paths
From paths to lazy paths
\[ Z_n(1; 1, 1, 1, 0; 0) \]

- \( Z_n(1; 1, 1, 1, 0; 0) \) is the number of lazy paths of size \( n \)
Definitions

Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications

$Z_n(1;1,1,1,0;0)$

- $Z_n(1;1,1,1,0;0)$ is the number of lazy paths of size $n$
- The number of $\gamma$ labels is the number of negative factors
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications

$Z_n(1; 1, 1, 1, 0; 0)$ is the number of lazy paths of size $n$

- The number of $\gamma$ labels is the number of negative factors
- The numbers of rows indexed by $\alpha/\gamma$ is the length of the initial maximal sequence of $(1, 1)$ steps
$Z_n(1; 1, 1, 1, 0; 0)$

- $Z_n(1; 1, 1, 1, 0; 0)$ is the number of lazy paths of size $n$.
- The number of $\gamma$ labels is the number of negative factors.
- The numbers of rows indexed by $\alpha/\gamma$ is the length of the initial maximal sequence of $(1, 1)$ steps.
- The number of columns indexed by $\beta$ is the number of $(2, 0)$ steps.
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications

$Z_n(1; 1, 1, 1, 0; 0)$

- $Z_n(1; 1, 1, 1, 0; 0)$ is the number of lazy paths of size $n$
- The number of $\gamma$ labels is the number of negative factors
- The numbers of rows indexed by $\alpha/\gamma$ is the length of the initial maximal sequence of (1, 1) steps
- The number of columns indexed by $\beta$ is the number of (2, 0) steps
- The number of $\alpha$, the number of $\beta$ are also known
Conclusion

- There is a simple insertion algorithm for LTLT’s
- They are natural and easy to use
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions
Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications
$Z_n(1; \alpha, \beta, \gamma, \delta; 1)$
$Z_n(\gamma; 1, 1, 0, 1; 1)$
A bijective proof for $Z_n(1; 1, 1, 1, 0; 0)$

To come

• Use the tree structure to prove $Z_n(\alpha, \beta, \gamma, \delta, q) = Z_n(\beta, \alpha, \gamma, \delta, q)$
• Find a weight on LTLT’s to use it in the general case of the PASEP
The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

Definitions

Staircase tableaux
Labeled tree-like tableaux
The insertion algorithm

Combinatorial applications

\[ Z_n(1; \alpha, \beta, \gamma, \delta; 1) \]
\[ Z_n(y; 1, 1, 0, 1; 1) \]
A bijective proof for
\[ Z_n(1; 1, 1, 0; 0) \]

Thank you!