

Definitions

Staircase tableaux

Labeled tree-like
tableaux

The insertion algorithm

Combinatorial applications

$Z_n(1; \alpha, \beta, \gamma, \delta; 1)$

$Z_n(y; 1, 1, 0, 1; 1)$

A bijective proof for

$Z_n(1; 1, 1, 1, 0; 0)$

The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

LaBRI, LIAFA

June 25, 2012

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- The PASEP : particles on a one-dimensional lattice

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- hop to the left/right

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- The PASEP : particles on a one-dimensional lattice
- hop to the left/right
- enter/exit from the left/right
- a Markov chain

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- The PASEP is studied through objects called staircase tableaux
- Labeled tree-like tableaux reveal an underlying tree structure in staircase tableaux
- Combinatorial properties

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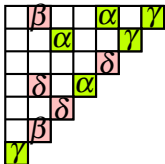
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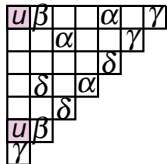
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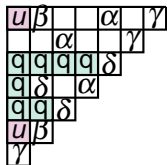


To fill the remaining boxes

- On the left of a β there is a u .
- On the left of a δ there is a q .
- Above an α or a delta δ there is a u .
- Above a β or a γ there is a q .

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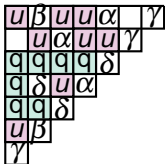


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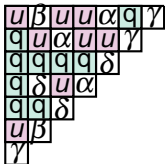


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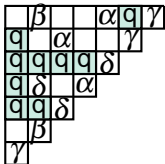


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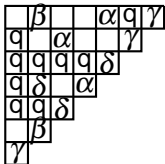
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- $\text{wt}(T) = \alpha^3 \beta^2 \gamma^3 \delta^3 q^9$
- $Z_n(\alpha, \beta, \gamma, \delta, q) = \sum_{T \in T_n} \text{wt}(T)$
- $Z_n(y; \alpha, \beta, \gamma, \delta, q) = \sum_{T \in T_n} \text{wt}(T) y^{t(T)}$

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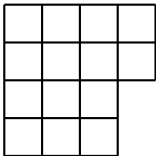
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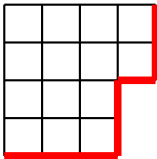
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Border edges

Labeled tree-like tableaux

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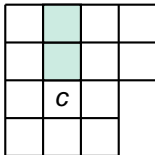
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$L_D(c)$

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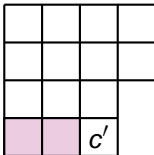
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$A_D(c')$

Labeled tree-like tableaux

Labeled tree-like tableau

A labeled tree-like tableau T of size n is a Ferrers diagram of half-perimeter $n + 1$ such that some boxes and all border edges are labeled with $1, \alpha, \beta, \gamma, \delta$, and satisfying the conditions :

- the Northwestern-most box (root box) is labeled by 1 ;
- the labels in the first row and the first column are the only labels 1 ;
- in each row and column, there is at least one labeled box;
- for each label α or γ , all boxes in $L_T(c)$ are empty and at least one box in $A_T(c)$ is labeled;
- for each label β or δ , all boxes in $A_T(c)$ are empty and at least one box in $L_T(c)$ is labeled.

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A labeled tree-like tableau

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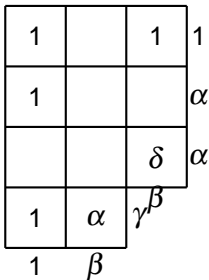
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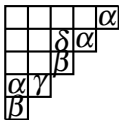
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Proposition

For $n \geq 2$, staircase tableaux of size $n - 1$ are in bijection with LTLTs of size n .



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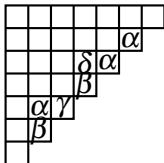
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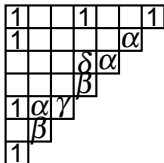
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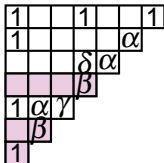
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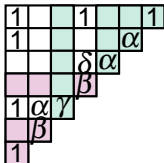
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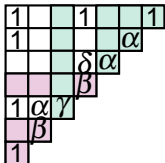
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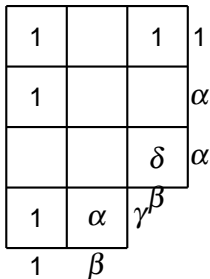
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\Leftrightarrow



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A bijection

The insertion algorithm induces a bijection between

- LTLTs of size n together with the choice of a border edge and a compatible bi-label
- LTLTs of size $n + 1$

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Row/column addition

1		1	1
1			α
		δ	α
1	α	γ	β
1	β		

(x, y)

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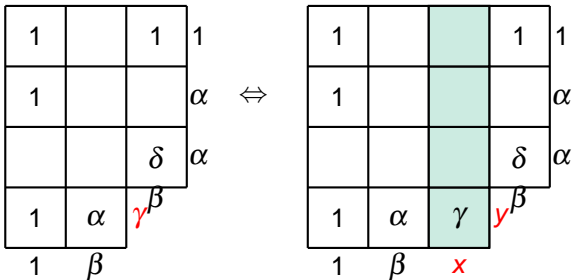
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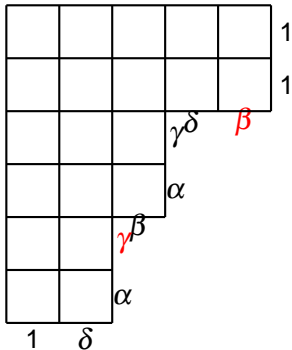
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Ribbon insertion

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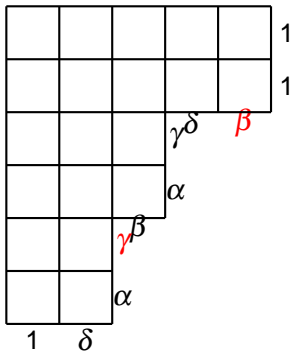
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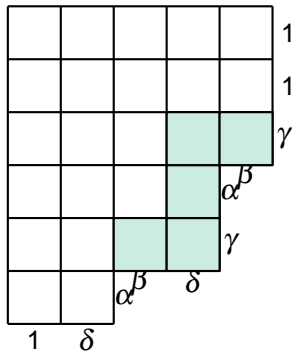
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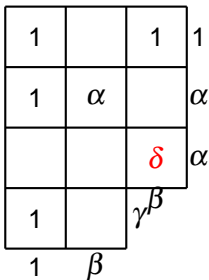
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The special box

Definition

The special box of a LTLT is the Northeastern-most labeled box among those that occur at the bottom of a column.



The insertion algorithm

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Require : a LTLT T of size n , one of its border edges e , a compatible bi-label (x, y) .

- 1 Find the special box s of T .
- 2 Add a row/column to T at edge e with new bi-label (x, y) .
- 3 If e is to the left of s , perform a ribbon addition between e and s .

Ensure : A final LTLT T' of size $n + 1$.

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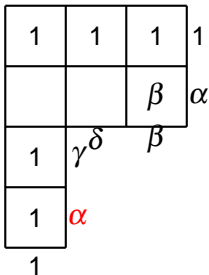
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The insertion algorithm : an example



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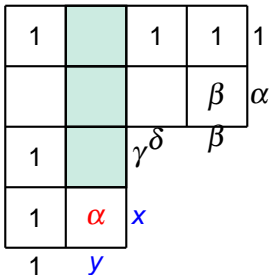
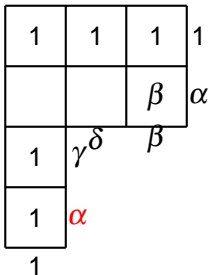
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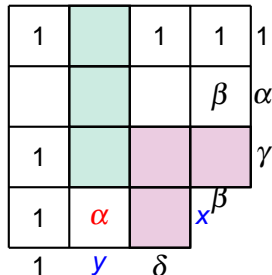
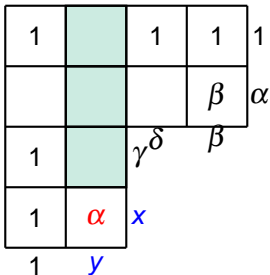
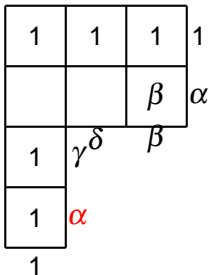
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A product formula for $Z_n(1; \alpha, \beta, \gamma, \delta; 1)$

$$Z_n(1; \alpha, \beta, \gamma, \delta; 1) = \prod_{j=0}^{n-1} (\alpha + \beta + \gamma + \delta + j(\alpha + \gamma)(\beta + \delta))$$

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Proposition

The polynomial $Z_n(y; 1, 1, 0, 1; 1)$ has all its roots in the segment $] -1, 0[$; it is stable and log-concave.

$$T(n, k) = \#\{\text{T of size } n, \text{ without } \gamma, \text{ having } k \alpha/\delta \text{ on the diagonal}\}$$
$$\text{Let } P_n(y) = \sum_{k \leq n} T(n, k) y^k$$

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Let $P_n(y) = \sum_{k \leq n} T(n, k) y^k$

Then

$$P_n(y) = (1 + (n+1)y + (n-1)y^2)P_{n-1}(y) + (y - y^3)P'_{n-1}(y)$$

Study of $Z_n(y; 1, 1, 0, 1; 1)$

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$$\text{Let } Q(y, t) = \sum_n P_n(y) \frac{t^n}{n!}, \text{ then } Q(y, t) = \left(\frac{y-1}{2y-(y+1)e^{t(y-1)}} \right)^{3/2} e^{\frac{t(y-1)}{2}}$$

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$$Z_n(1; 1, 1, 1, 0; 0)$$

What is the number of staircase tableaux with no qs nor δs ?

A lazy path

Definition

A lazy path of size n is a path on the lattice $\mathbb{Z} \times \mathbb{Z}$

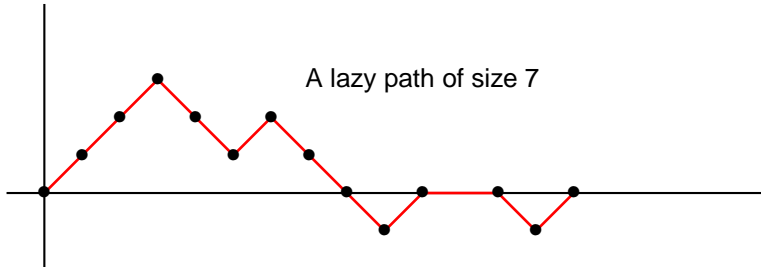
- starting at $(0, 0)$, ending at $(2n, 0)$
- whose steps are $(1, 1)$, $(1, -1)$ or $(2, 0)$
- such that its steps $(2, 0)$ only appear on the x-axis

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From staircase tableaux to trees

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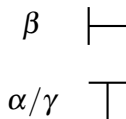
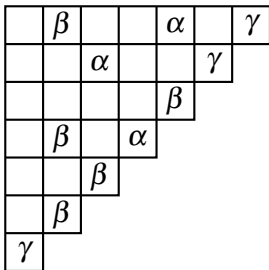
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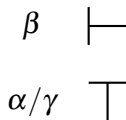
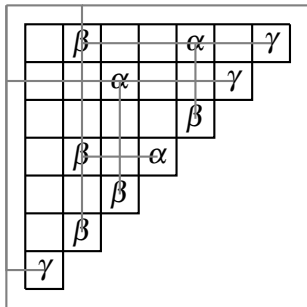
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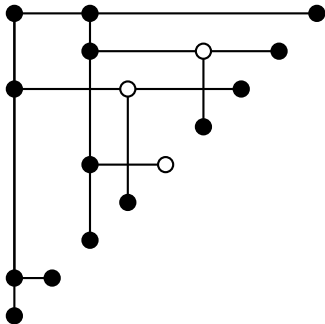
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$$\beta \quad \vdash$$

$$\alpha/\gamma \quad \top$$

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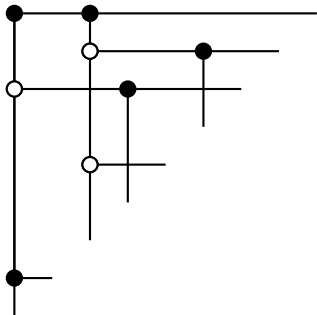
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$$\beta \quad \vdash$$
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The tree structure in staircase tableaux

J.-C. Aval, A. Boussicault, S. Dasse-Hartaut

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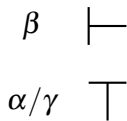
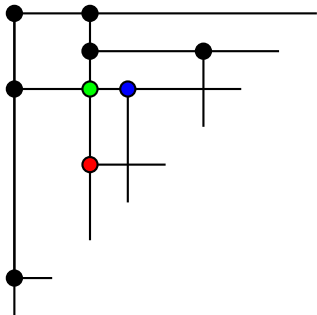
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A bijective proof for $Z_n(1; 1, 1, 1, 0; 0)$

$$\begin{array}{c} q \\ \gamma \end{array} X \Leftrightarrow \begin{array}{c} \text{nodes of left} \\ \text{depth more than 1} \end{array}$$

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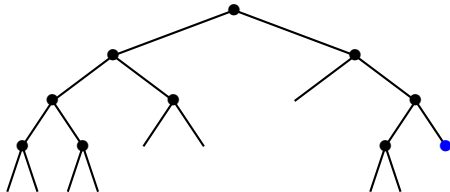
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A bijective proof for $Z_n(1; 1, 1, 1, 0; 0)$



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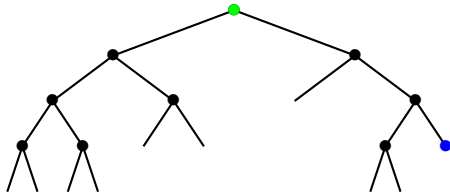
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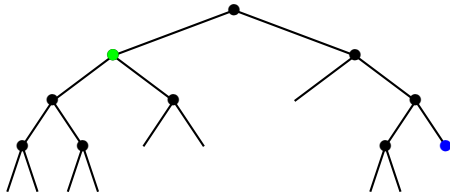
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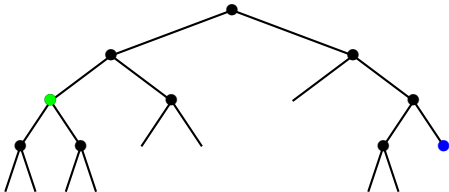
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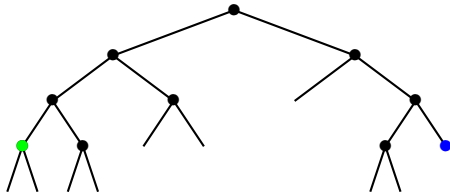
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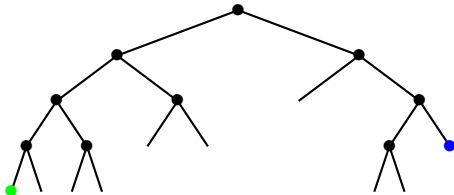
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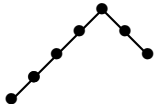
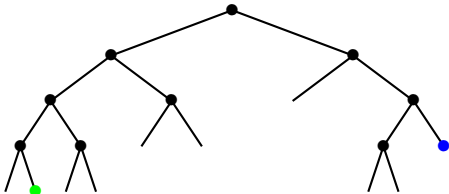
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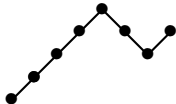
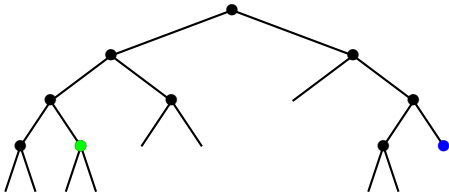
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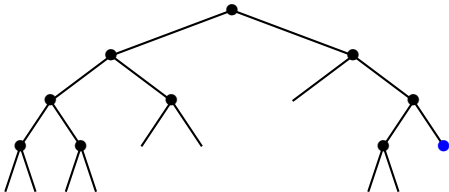
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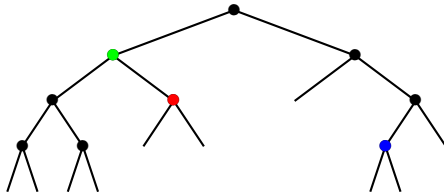
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From paths to lazy paths



From paths to lazy paths

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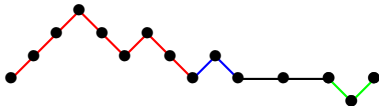
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$$Z_n(1; 1, 1, 1, 0; 0)$$

- $Z_n(1; 1, 1, 1, 0; 0)$ is the number of lazy paths of size n

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$$Z_n(1; 1, 1, 1, 0; 0)$$

- $Z_n(1; 1, 1, 1, 0; 0)$ is the number of lazy paths of size n
- The number of γ labels is the number of negative factors

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$$Z_n(1; 1, 1, 1, 0; 0)$$

- $Z_n(1; 1, 1, 1, 0; 0)$ is the number of lazy paths of size n
- The number of γ labels is the number of negative factors
- The numbers of rows indexed by α/γ is the length of the initial maximal sequence of $(1, 1)$ steps

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$$Z_n(1; 1, 1, 1, 0; 0)$$

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- The number of γ labels is the number of negative factors
- The numbers of rows indexed by α/γ is the length of the initial maximal sequence of $(1, 1)$ steps
- The number of columns indexed by β is the number of $(2, 0)$ steps

$$Z_n(1; 1, 1, 1, 0; 0)$$

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- $Z_n(1; 1, 1, 1, 0; 0)$ is the number of lazy paths of size n
- The number of γ labels is the number of negative factors
- The numbers of rows indexed by α/γ is the length of the initial maximal sequence of $(1, 1)$ steps
- The number of columns indexed by β is the number of $(2, 0)$ steps
- The number of α , the number of β are also known

J.-C. Aval, A.
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- There is a simple insertion algorithm for LTLT's
- They are natural and easy to use

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To come

- Use the tree structure to prove
$$Z_n(\alpha, \beta, \gamma, \delta, q) = Z_n(\beta, \alpha, \gamma, \delta, q)$$
- Find a weight on LTLT's to use it in the general case of the PASEP

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Thank you!